A New Algorithm for Creating DEMs with Smooth Elevation Profiles

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1. Introduction

Plots of longitudinal elevation profiles generated from a DEM and a corresponding D8 flow grid generally show an unrealistic step-like character, with long segments of slope zero separated by abrupt steps with steep slopes. This occurs as a result of limited vertical and horizontal accuracy in the DEM, particularly for DEMs with integer-valued elevations and a vertical resolution of one meter or one foot. The essence of the problem is that slopes on streamline elevation profiles are typically very small, on the order of 0.0001 or less. However, if the vertical resolution is Δz and the horizontal resolution is Δx, then the minimum, nonzero slope that is resolvable between two adjacent pixels is close to Δz / Δx. So, for example, if the vertical resolution is one meter and the horizontal resolution is 10 meters, slopes less than about 0.1 will be unresolvable and will usually get mapped to a value of zero. Note that even for a vertical resolution of 1 centimeter, the minimum resolvable value would be 0.001, still too large to resolve the actual along-channel slope. This issue becomes a real problem in spatially-distributed hydrologic models that use DEM-derived channel slope to compute flow velocity, v, from Manning's formula

$$v = (1/n)\, R_h^{\,2/3} S^{1/2}. \tag{1}$$

and the kinematic wave approximation. Here, n is the Manning's roughness parameter and R_h is the hydraulic radius.

When looking at a plot of a single elevation profile, it is clear that we want to apply some kind of smoothing or curve-fitting operation that replaces the elevation values on the jagged, original profile with a new, smoother set of values. Moreover, assuming that the original values are accurate to within the upper and lower bounds that are set by the vertical resolution, it seems reasonable that after rounding the new values to the same resolution we should recover the original values, if possible. What is not immediately clear is what operation we can perform on the original DEM so that all of the streamline elevation profiles will get smoothed in this way, without altering any of the original D8 flow directions.

2. A "Profile-smoothing" Algorithm

The purpose of this paper is to present one solution to this problem that is conceptually appealing and that seems to work relatively well. The idea is to first assume that Flint's Law is approximately valid over the entire DEM. Flint's Law (see all cited references) is an empirical relationship that expresses local channel slope, S, as a power-law function of basin contributing area

$$S = c A^{\theta}. \tag{2}$$

The exponent, θ, is sometimes called the concavity. Using a D8 flow grid, one then identifies the set of grid cells that lie on the streamline of the main channel in the basin of interest. Recall that the main channel is typically identified using a grid of contributing areas (computed by the D8 method) and repeatedly stepping upstream toward the D8 neighbor cell with the largest contributing area until a drainage divide is reached. Let z_0 denote the elevation of the grid cell to which the main channel's first grid cell flows. According to Flint's Law, the predicted elevation for the k^{th} grid cell on the main channel is then

$$z'_k(c,\theta) = z_0 + c\sum_{j=1}^{k} A_j^{\theta}\,\Delta L_j \quad (3)$$

where A_j is the contributing area of the j^{th} grid cell on the main channel and ΔL_j is the horizontal distance between adjacent main-channel grid cells. A nonlinear least-squares regression procedure is then used to estimate the parameters c and θ in equation (3) that give the best fit to the main channel elevation values, z_k. If we assume that the same parameters c and θ are approximately valid for every other elevation profile in the DEM, we can then use them to compute a new grid of channel slopes from the values in the contributing area grid. This grid of channel slopes is guaranteed to decrease smoothly downstream since contributing areas computed by the D8 method always increase downstream. The final step is to modify the original elevation values so that the slope computed between every grid cell and its downstream neighbor is exactly equal to the value predicted by the new channel slope grid. We do this by using an iterative procedure, starting with the grid cells that are furthest downstream and then computing and applying the small, floating-point elevation changes that must be made to upstream neighbor cells in order to achieve the prescribed channel slope. The iteration continues upstream until every grid cell has the prescribed slope. Figure 1 shows the result of applying this procedure to the main channel of Beaver Creek, Kentucky. The smooth curve provides much better estimates of channel slope but sometimes results in fairly large differences in elevation.

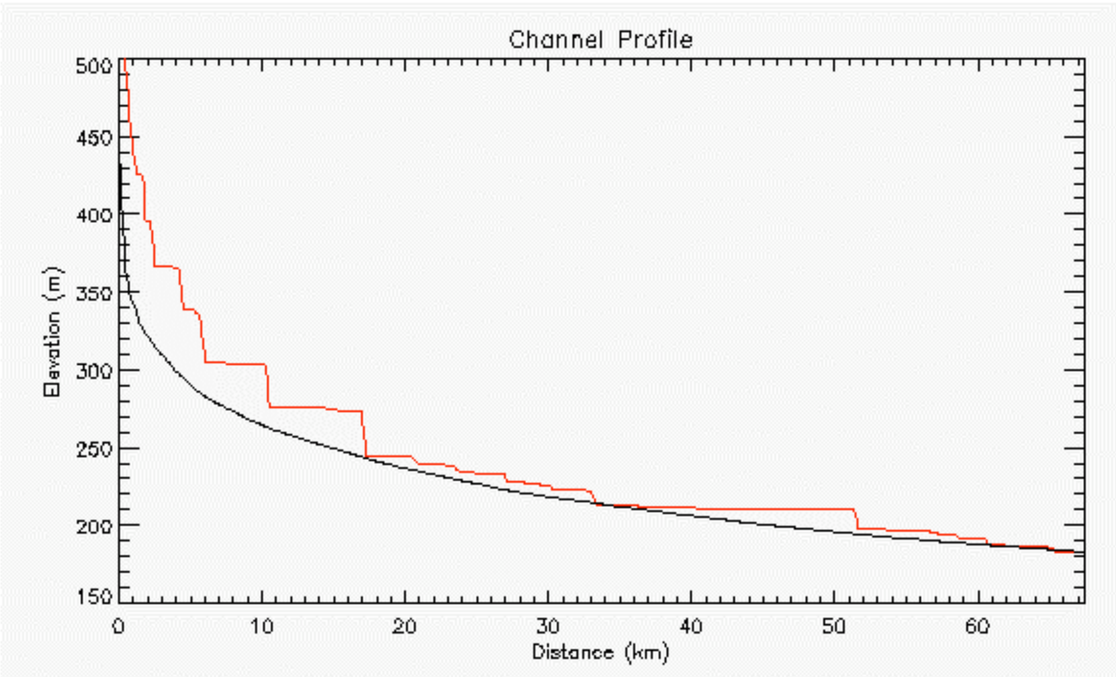

Figure 1. Comparison of main-channel elevation profiles before and after the profile-smoothing operation.

As explained previously, limited vertical and horizontal accuracy in DEMs can result in enormous errors in channel slope, S, as measured between adjacent grid cells. This is especially true for DEMs in which elevations have been rounded to the nearest foot or meter. As already illustrated, computed slopes can differ from actual slopes by a factor of 10,000 or more. However, contributing area, A, measured from a DEM depends only on the horizontal resolution and therefore the relative error in A is very small for basins that are much larger than the grid cell size. It follows that even if Flint's Law is only a crude approximation, using it to compute channel slopes from areas is likely to be more accurate than measuring slopes between grid cells in the DEM. Also, because of the inverse relationship between S and A, the relative error in A is smallest for the larger basins where the measured error in S is largest. This approach therefore allows us to trade large measurement errors in S for small measurement errors in A.

3. Conclusions

The fact that this algorithm works relatively well (at least for the fairly mature and homogeneous fluvial landscapes for which it has been tested) is somewhat surprising and points to an organizing principle in real landscapes that is not yet well-understood. Perhaps most surprising is the fact that best-fit values of c and θ obtained for the main channel produce reasonable results when applied to the landscape as a whole. In test cases the value of θ tends to be close to -0.55. This is also the average value reported by Whipple (2004). It is also about what one would expect based on combining the empirical slope-discharge equation of hydraulic geometry with an exponent close to -0.5 and a discharge-area power-law with an exponent close to 1. Note, however, that Flint's Law is not expected to apply to the concave down portion of a longitudinal profile near a drainage divide. As a result of this, the algorithm tends to produce elevations near drainage divides that are sharper than in the original DEM. However, the author has found using DEMs of different resolutions for Beaver Creek, Kentucky that the modified DEM was actually in better agreement with a higher-resolution DEM for the same area than the original DEM.

This basic "profile-smoothing" algorithm can be modified in various ways. For example, the slope value predicted from Flint's Law can be rejected if it results in an elevation change that is greater than the vertical resolution of the original DEM. One could also use all of the DEM grid cells in the nonlinear regression to estimate c and θ, instead of only those on the main channel. Work is ongoing to refine the algorithm and to make it as robust as possible.

References

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